# Kulkarni's Academy of Mechanical Engineering

# MECHANICAL ENGINEERING

## Solutions with Explanation

Morning Session: 4 Feb. 2017 (09:00 AM to 12:00 Noon)

GATE 2017



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**01.** A motor driving a solid circular steel shaft transmits 40 kW of power at 500 rpm. If the diameter of the shaft is 40mm, the maximum shear stress in the shaft is ___MPa.

**Answer:** 60.79 MPa

**Explanation:**

$$P = T \times W$$

$$40 \times 10^3 = T \times \frac{2\pi \times 500}{60} = 763.9\ \text{Nm} = 7639 \times 10^3\ \text{Nmm}$$

$$\frac{T}{J} = \frac{\tau}{R} = \tau = \frac{TR}{J} = \frac{763.9 * 10^3 * 20 * 32}{\pi * 40^4}$$

$$\tau_{max.} = 60.79\ \text{N/mm}^2 = 60.79\ \text{MPa}$$

**02.** Cylindrical pins of diameter $15^{\pm 0.020}$ mm are being produced on a machine. Statistical quality control tests show a mean of 14.995 mm and standard deviation of 0.004 mm. The process capability index $C_p$ is

a) 0.833 b) 1.667 c) 3.333 d) 3.750

**Answer:** b

**Explanation:**

$$C_p = \frac{U - L}{6\sigma}$$

$$U = 15.02$$

$$L = 14.98$$

$$\sigma = 0.004$$

$$C_p = \frac{15.02 - 14.98}{6 * 0.004} = 1.667$$

**03.** The damping ratio for a viscously damped spring mass system, governed by the relationship $m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$, is given by

a) $\sqrt{\frac{c}{mk}}$ b) $\frac{c}{2\sqrt{km}}$

c) $\frac{c}{\sqrt{km}}$

d) $\sqrt{\frac{c}{2mk}}$

Answer: b

Explanation:

Damping ratio $= \frac{C}{2\sqrt{Km}}$

04. Match the process with their characteristics.

| Process | | Characteristics | |
|---|---|---|---|
| P | Electrical Discharge machining | 1. | No residual stress |
| Q | Ultrasonic machining | 2. | Machining of electrically conductive materials |
| R | Chemical machining | 3. | Machining of glass |
| S | Ion Beam machining | 4. | Nano- machining |

a) $P-2, Q-3, R-1, S-4$

b) $P-3, Q-2, R-1, S-4$

c) $P-3, Q-2, R-4, S-1$

d) $P-2, Q-4, R-3, S-1$

Answer: a

05. In an arc welding process, welding speed is doubled. Assuming all other process parameters to be constant, the cross sectional area of the weld bead will

a) Increase by 25%

b) Increase by 50%

c) Reduce by 25%

d) Reduce by 50%

Answer: d

Explanation:

Metal deposit rate (MDR) = Area (A) x Weld Speed (V)

= Constant

$A_1V_1 = A_2V_2 \quad ; \; V_2 = 2V_1 \text{(given)}$

$$A_1V_1 = A_2(2V_1) \Rightarrow A_2 = \frac{A_1}{2}$$

Reduce by half (50%)

06. Consider the two-dimensional velocity field given by $\vec{V} = (5 + a_1x + b_1y)\hat{i} + (4a_2x + b_2y)\hat{j}$, where $a_1, b_1, a_2$ and $b_2$ Are constants. Which one of the following conditions needs to be satisfied for the flow to be incompressible?

a) $a_1 + b_1 = 0$ b) $a_1 + b_2 = 0$

c) $a_2 + b_2 = 0$ d) $a_2 + b_1 = 0$

Answer: b

Explanation:

$$\vec{V} = u\hat{i} + \upsilon\hat{j}$$

$$u = 5 + a_1x + b_1y; \upsilon = 4 + a_2x + b_2y$$

Every fluid flow must satisfy its corresponding mass conservation (continuity) equation

For a 2 – dimensional incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{dy} = 0 \Rightarrow a_1 + b_2 = 0$$

07. Consider a beam with circular cross-section of diameter d. The ratio of the second moment of area about the neutral axis to the section modulus of the area is

a) $\frac{d}{2}$ b) $\frac{\pi d}{2}$ c) d d) $\pi d$

Answer: a

Explanation:

Second moment of area (neutral axis), $I = \frac{\pi d^4}{64}$

Section modulus $= \frac{I}{y_{max}}$

$$\text{ratio} = \frac{I}{\frac{I}{y_{max}}} = y_{max}$$

For a circular section, $y_{max} = r = \frac{d}{2}$

08. A particle of unit mass is moving on a plane. Its trajectory, in polar coordinates, is given by $r(t) = t^2, \theta(t) = t,$ where t is time. The kinetic energy of the particle at time t = 2 is

a) 4 b) 12 c) 16 d) 24

Answer: c

Explanation:

$$V_t = rw = t^2 \frac{d\theta}{dt}; V_r = \frac{dr}{dt} = 2t = 2 \times 2 = 4$$

$$V_t = 2^2 \times 1 = 4; V_r = 4$$

$$V = \sqrt{V_r^2 + V_t^2} = \sqrt{4^2 + 4^2} = \sqrt{32}$$

$$\text{K.E.} = \frac{1}{2} mV^2 = \frac{1}{2} * 1 * \left(\sqrt{32}\right)^2 = 16$$

09. Water (density = 1000 kg/m$^3$) at ambient temperature flows through a horizontal pipe of uniform cross section at the rate of 1 kg/s. If the pressure drop across the pipe is 100 kPa, the minimum power required to pump the water across the pipe, in watts, is_____

Answer: 100

Explanation:

Power = w Q $h_L$

(Power is required to overcome losses, $h_L$)

$$h_L = \frac{\Delta P}{w} = \frac{100 \times 10^3}{9810} = 10.1936\,m$$

$$\dot{m} = PQ \Rightarrow Q = \frac{\dot{m}}{\rho} = \frac{1}{10^3} = 10^{-3}\,Kg/S$$

$$\text{Power} = 9810 \times 10^{-3} \times 10.1936 = 100\,W$$

10. In the engineering stress-strain curve for mild steel, the Ultimate Tensile Strength (UTS) refers to

a) Yield stress b) Proportional limit c) Maximum stress d) Fracture stress

Answer: c

Explanation:

For mild steel, from engineering stress strain diagram the UTS represents maximum stress.

11. Consider the following partial differential equation for $u(x,y)$ with the constant $c > 1$.

Solution of this equation is $\frac{\partial u}{\partial y}+c\frac{\partial u}{\partial x}=0$

a) $u(x,y)=f(x+cy)$ b) $u(x,y)=f(x-cy)$

c) $u(x,y)=f(cx+y)$ d) $u(x,y)=f(cx-y)$

Answer: b

Explanation:

We can verify from options

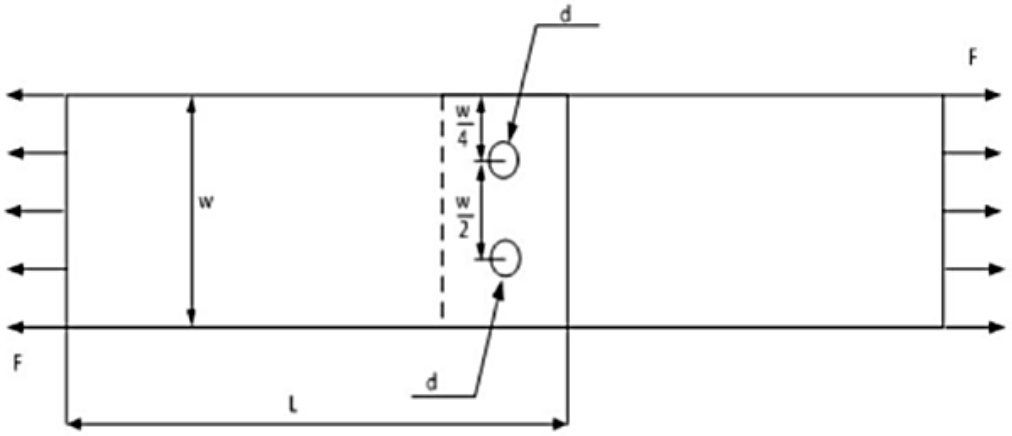
$u(x,y)=f(x-cy)$

$\frac{\partial u}{\partial y}+C\frac{\partial u}{\partial x}=f'(x-cy)(-c)+cf'(x-cy)=0$

The solution of $\frac{\partial u}{\partial y}+C\frac{\partial u}{\partial x}=0$ is

$U(x,y)=f(x-cy)$

12. Consider the schematic of a riveted lap joint subjected to tensile load F, as shown below. Let d be the diameter of the rivets, and $S_f$ be the maximum permissible tensile stress in the plates. What should be the minimum value for the thickness of the plates to guard against tensile failure of the plates? Assume the plates to be identical.




a) $\frac{F}{S_f(W-2d)}$

b) $\frac{F}{S_f W}$

c) $\frac{F}{S_f (W-d)}$

d) $\frac{2F}{S_f W}$

Answer: a

Explanation:

Net resisting area = (W- 2d) t

$F = S_f (W-2d)t$

$t = \frac{F}{S_f(W-2d)}$

13. Metric thread of 0.8 mm pitch is to be cut on a lathe. Pitch of the lead screw is 1.5 mm. If the spindle rotates at 1500 rpm, the speed of rotation of the lead screw (rpm) will be_____

Answer: 800

Explanation:

Pitch = 0.8 mm

Lead screw pitch = 1.5 mm

Spindle speed = 1500rpm

In one rotation of lead, it should advance 0.8 mm.

Fraction of revolution in one rotation of spindle

$= \frac{0.8}{1.5}$

Rpm of lead $= \frac{0.8}{1.5} \times 1500 = 800\,\text{rpm}$

14. The differential equation

$\frac{d^2y}{dx^2} + 16y = 0$ For $y(x)$ with the two boundary conditions $\left.\frac{dy}{dx}\right|_{x=0} = 1$ and $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = -1$ has

a) No solution

b) Exactly two solutions

c) Exactly one solution

d) infinitely many solutions

Answer: a

Explanation:

The given different equation is homogenous second order differential equation, the complete solution is

y = complementary function

$(D^2 + 16)y = 0 \Rightarrow D^2 + 16 = 0$

$D \pm 4i$

$y = c_1 \cos 4x + c_2 \sin 4x$

$\frac{dy}{dx} = -4c_1 \sin 4x + 4c_2 \cos 4x$

Boundary condition (1) $\Rightarrow 4c_2 = 1 \Rightarrow c_2 = \frac{1}{4}$

$BC(2) \Rightarrow c_2 = \frac{1}{4}$

Therefore, for given boundary conditions, the given differential equation has no solution.

15. For steady flow of a viscous incompressible fluid through a circular pipe of constant diameter, the average velocity in the fully developed region is constant. Which one of the following statements about the average velocity in the developing region is TRUE?

a) It increases until the flow is fully developed

b) It is constant and is equal to the average velocity in the fully developed region

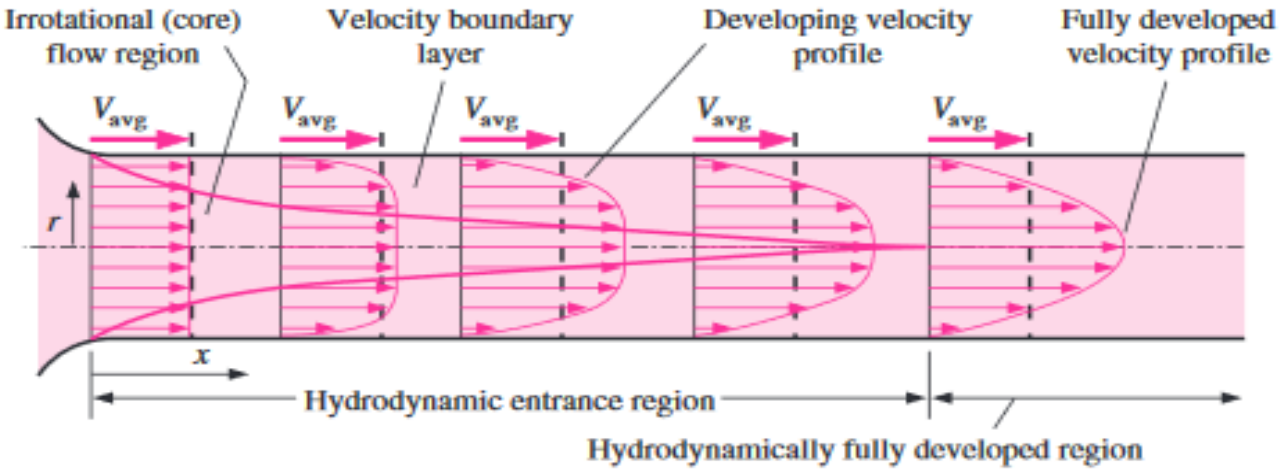
c) It decreases until the flow is fully developed

d) It is constant but is always lower than the average velocity in the fully developed region.

Answer: b

Explanation:

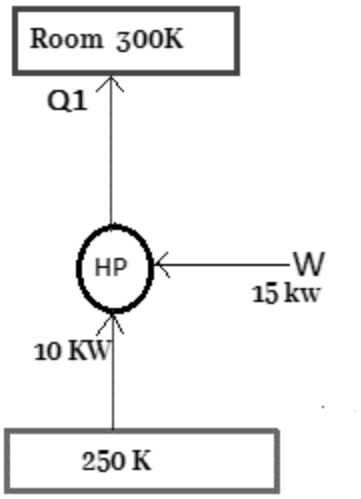
As the pipe is of uniform diameter and the flow is steady incompressible, therefore the discharge remains constant at every section. As the diameter is constant, the avg. velocity is constant everywhere (ref. figure)




16. A heat pump absorbs 10 kW of heat from outside environment at 250 K while absorbing 15 kW of work It delivers the heat to a room that must be kept warm at 300 K. The Coefficient of Performance (COP) of the heat pump is_____

Answer: 1.67

Explanation:




$$\text{CoP}_{np} = \frac{Q_1}{W} = \frac{25}{15} = 1.67$$

Note· COP of heat pump = $\frac{T_h}{T_h - T_L}$ is used only for REVERSIBLE heat pump.

$\text{CoP}_{np} = \frac{Q_1}{W}$ Can be used for ANY heat pump.

17. A six-face fair dice is rolled a large number of times. The mean value of the outcomes is_____

Answer: 3.5

Explanation:

| Valve on the die (x) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Probability P(X) | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

$$\text{Mean} = E(x) = \frac{1+2+3+4+5+6}{6} = 3.5$$

Since mean is same every time, required mean is 3.5

18. The product of eigenvalues of the matrix P is

$$P = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

a) – 6 b) 2 c) 6 d) – 2

Answer: b

Explanation:

From the property of eigenvalues

Product of eigenvalues = |P|

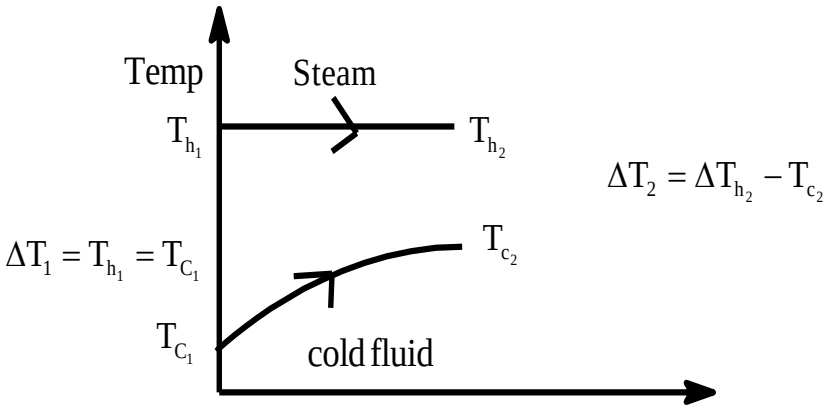
$$\begin{vmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{vmatrix} = 2(3-6)+1(8-0)$$

$$= 2$$

19. Saturated steam at 100°C condenses on the outside of a tube. Cold fluid enters the tube at 20°C and exits 50°C. The value of the Log Mean Temperature Difference (LMTD) is_____°C.

Answer: 63.83 °C

Explanation:




For phase change HX, LMID of parallel and counter flow Hx will be same

$$\Delta T_1 = T_{h_1} - T_{c_1} = 100 - 20 = 80$$

$$\Delta T_2 = T_{h2} - T_{c_2} = 100 - 50 = 50$$

$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{80-50}{\ln\left(\frac{80}{50}\right)}$$

$$LMTD = 63.829^0 C$$

20. Which one of the following is NOT a rotating machine?

a) Centrifugal pump
b) Gear pump
c) Jet pump
d) Vane pump

Answer: c

Explanation:

Centrifugal, gear and vane pumps have rotating parts.

21. The molar specific heat at constant volume of an ideal gas is equal to 2.5 times the universal gas constant (8.314 J/mol K). When the temperature increases by 100 K, the change in molar specific enthalpy is_____J/mol.

Answer: 2909.9

Explanation:

$C_p$ and $C_V$ are molar specific heats

$C_V = 2.5\vec{R}$

$\Delta T = 100K$

$\Delta H = C_p \Delta T$

$C_p - C_V = \vec{R}$

$\Rightarrow C_p = \vec{R} + C_V$

$C_p = 3.5\vec{R}$

$$\Delta H = 3.5\,\vec{R} * 100$$

$$\Delta H = 3.5 \times \frac{8.314}{mol - k} \times 100K$$

$$\Delta H = 2909.9 J / mol$$

22. The Poisson's ratio for a perfectly incompressible linear elastic material is

a) 1 b) 0.5 c) 0 d) Infinity

Answer: b

Explanation:

Volumetric strain is given by $\varepsilon_v = \frac{\Delta V}{V}$

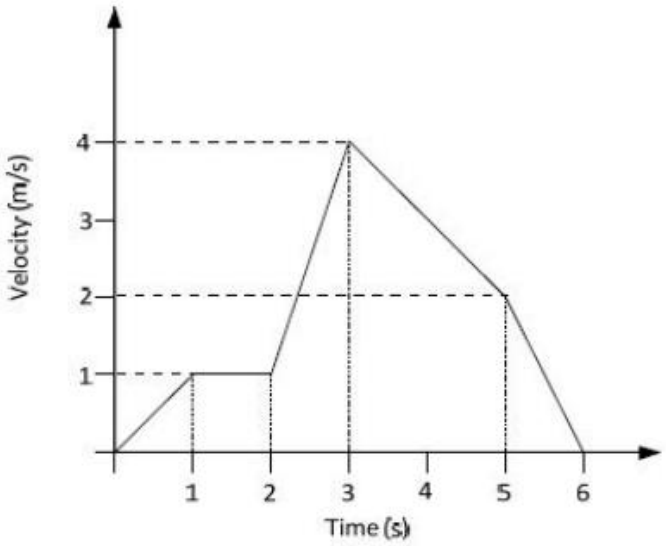
$$\varepsilon_V = \frac{(1-2\mu)}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$\mu =$ Poisson's ratio

For an incompressible material change in volume $(\Delta V) = 0 \Rightarrow \varepsilon_V = 0$

$$\Rightarrow 1-2\mu = 0 \Rightarrow \mu = \frac{1}{2} = 0.5$$

23. The following figure shows the velocity-time plot for a particle traveling along a straight line. The distance covered by the particle from t = 0 at t = 5s is_____m.




Answer: 10

Explanation:

$$\text{Distance} = \left(\frac{1}{2}\times 1\times 1\right) + (1\times 1) + \frac{1}{2}(5\times 1) + \left(\frac{1}{2}\times 6\times 2\right)$$

$$= 10 \text{ m}$$

24. In a metal forming operation when the material has just started yielding, the principle stresses are $\sigma_1 = +180\,\text{MPa}, \sigma_2 = -100\,\text{MPa}, \sigma_3 = 0$. Following von Mises criterion, the yield stress is_____MPa.

Answer: 245.76

Explanation:

According to von Mises theory, for yielding

$$\sigma_{yt} = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]}$$

$$\sigma_{yt} = \sqrt{\frac{1}{2}\left[\{180-(-100)\}^2+(-100-0)^2+(-180)^2\right]}$$

$$\sigma_{yt} = 245.76\,\text{MPa}$$

25. The value of

$$\lim_{x\to 0}\frac{x^3-\sin(x)}{x}$$ is

a) 0 b) 3 c) 1 d) -1

Answer: d

Explanation:

$$\underset{x\to 0}{Lt}\left(x^2-\frac{\sin x}{x}\right) = 0-1 = -1$$

26. Assume that the surface roughness profile is triangular as shown schematically in the figure. If the peak to valley height is $20\mu m$, the central line average surface roughness $R_a\,(\text{in}\,\mu m)$ is

a) 5 b) 6.67 c) 10 d) 20

Answer: a

Explanation:

$$CLA(Ra) = \frac{H_{max}}{4} = \frac{20}{4} = 5$$

27. Following data refers to the jobs (P, Q, R, S) which have arrived at a machine for scheduling. The shortest possible average flow time is_____days.

| Job | Processing Time (days) |
|---|---|
| P | 15 |
| Q | 9 |
| R | 22 |
| S | 12 |

Answer: 31

Explanation:

| Job | Processing | Flow time |
|---|---|---|
| Q | 9 | 9 |
| S | 12 | 21 |
| P | 15 | 36 |
| R | 22 | 58 |
| | | 124 |

$$\text{Average flow time} = \frac{124}{4} = 31$$

28. A parametric curve defined by

$x = \cos\left(\frac{\pi u}{2}\right), y = \sin\left(\frac{\pi u}{2}\right)$ in the range of $0 \le u \le 1$ is rotated about the X-axis by 360 degrees. Area of the surface generated is

a) $\frac{\pi}{2}$ b) $\pi$ c) $2\pi$ d) $4\pi$

Answer: c

29. Consider the matrix

$$P = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}.$$

Which one of the following statements about P is INCORRECT?

a) Determinant of P is equal to 1.

b) P is orthogonal

c) Inverse of P is equal to its transpose

d) All eigenvalues of P are real numbers

Answer: d

Explanation:

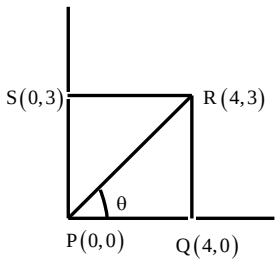
$PP^T = I \Rightarrow P^T = P^{-1}$

So, option d is incorrect statement

30. A rectangular region in a solid is in a state of plane stain. The (x, y) coordinates of the corners of the un-deformed rectangle are given by P (0, 0), Q (4, 0), R (4, 3) S (0, 3). The rectangle is subjected to uniform strains, $\varepsilon_{xx} = 0.001, \varepsilon_{yy} = 0.002, \gamma_{xy} = 0.003$. The deformed length of the elongated diagonal upto three decimals places, is_____units.

Answer: 5.014

Explanation:




$PR = \sqrt{4^2+3^2} = 5$

$PR = \sqrt{4^2+3^2} = 5$

Strain in diagonal $(PR) = \dfrac{\Delta PR}{PR}$

$\in_{xx} \cos^2\theta + \in_{yy} \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$

$0.001\times\left(\frac{4}{5}\right)^2 + 0.002\left(\frac{3}{5}\right)^2 + 0.003\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$2.8\times10^{-3}$

$\dfrac{\Delta PR}{PR} = 2.8\times10^{-3}$

$\Delta PR = 2.8\times10^{-3}\times5 = 0.014\,mm$

Deformed length of diagonal $PR = 5+0.014 = 5.014\,mm$

31. For a steady flow, the velocity field is $\vec{V} = \left(-x^2+3y\right)\hat{i} + \left(2xy\right)\hat{j}$. The magnitude of the acceleration of a particle at (1, -1) is

a) 2 b) 1 c) $2\sqrt{5}$ d) 0

Answer: c

Explanation:

$\vec{V} = \left(-x^2+3y\right)\hat{i} + 2xy\hat{j}$

$u = -x^2+3y; \upsilon = 2xy$

$a_x = u\dfrac{\partial u}{\partial x} + \upsilon\dfrac{\partial u}{\partial y} = \left(-x^2+3y\right)\left(-2x\right) + 2xy\left(3\right)$

$a_x = \left[-(1)^2+3(-1)\right](-2(1)) + 6(1)(-1)$

$a_1 = (-4)(-2) - 6$

$$a_x = 2$$

$$a_y = u\frac{\partial \upsilon}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial y} = \left(-x^2 + 3y\right)(2y) + (2xy)(2x)$$

$$a_y = \left[(1)^2 + 3(-1)\right]\left[2(-1) + 4(1)(-1)\right]$$

$$a_y = (-4)(-2) - 4$$

$$ay = 4$$

$$a = \sqrt{a_x^2 + {}_y^2} = \sqrt{2^2 + 4^2} = \sqrt{20}$$

$$= 2\sqrt{5}$$

32. Moist air is treated as an ideal gas mixture of water vapour and dry air (molecular weight of air = 28. 84 and molecular weight of water = 18). At a location, the total pressure is 100 KPa, the temperature is $30^0$C and the relative humidity is 55%. Given that the saturation pressure of water at $30^0$C is 4246 Pa, the mass of water vapour per kg of dry air is_____grams.

Answer: 14.923

Explanation:

$$\phi = \frac{P_V}{P_{V_s}} \Rightarrow 0.55 = \frac{P_V}{4.246} \Rightarrow P_v = 2.3353\,\text{KPa}$$

$$w = \frac{18}{28.84}\left(\frac{2.3353}{100 - 2.3353}\right) = 0.014923\,\text{KgV / Kgda}$$

$$= 14.923\,\text{gmv / Kg da}$$

33. Air contains 79% $N_2$ and 21% $O_2$ on a molar basis. Methane ($CH_4$) is burned with 50% excess air than required stoichiometrically. Assuming complete combustion of methane, the molar percentage of $N_2$ in the products is) _____.

Answer: 73.83%

Explanation:

100 Liter air contains 79 liter of $N_2$ and 21 liter of $O_2$

The combustion equation with 50% excess air is

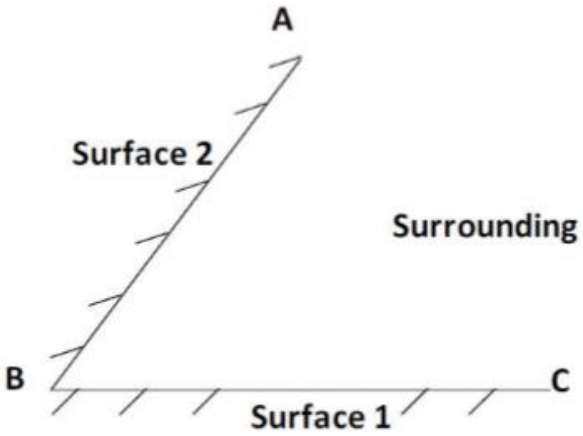
$$CH_4 + 1.5\times 2\left[3.762N_2 + O_2\right] \rightarrow 2H_2O + CO_2 + 3\times 3.762\,N_2 + O_2$$

It is assumed that nitrogen is inert and does not participate in the reaction. Whatever nitrogen is there in reactants, same will come out in the products.

$$N_2 = \frac{3\times 3.762}{2+1+3\times 3.762+1}\left(\text{vol or molar basis}\right)$$

$$= 0.7383 = 73.83\%$$

34. Two black surfaces, AB and BC, of lengths 5 m and 6 m, respectively, are oriented as shown. Both surfaces extend infinitely into the third dimension. Given that view factor $F_{12} = 0.5, T_1 = 800\,K,\quad T_2 = 600\,K,\ T_{surrounding} = 300\,K$ and Stefan Boltzmann constant, $\sigma = 5.67\times 10^{-8}\,W/\left(m^2K^4\right)$, the heat transfer rate from Surface 2 to the surrounding environment is _____kW.




Answer: 14.69 KW

Explanation:

From Summation rule,

$$F_{21} + F_{22} + F_{23} = 1 \quad ; F_{22} = 0$$

$$F_{21} + F_{23} = 1$$

$$A_1 F_{12} = A_2 F_{21}$$

$$6 \times 0.5 = 5 \times F_{21}$$

$$F_{21} = 0.6$$

$$F_{23} = 1 - F_{21} = 0.4$$

In this Question, the rate of heat transfer from surface 2 to surroundings (Surface 3) is asked (Net heat transfer between 2 and 3 is not asked in this problem) therefore

$$Q_{23} = \sigma A_2 F_{23} T_2^4$$

$$= 5.67 \times 10^{-8} \times 5 \times 1 \times 600^4 \times 0.4 = 14696.64 \text{ W}$$

$$= 14.69 \text{ KW}$$

35. Consider steady flow of an incompressible fluid through two long and straight pipes of diameters $d_1$ and $d_z$ arranged in series. Both pipes are of equal length and the flow is turbulent in both pipes. The friction factor for turbulent flow though pipes is of the form, $f = K(Re)^{-n}$, where K and n are known positive constants and Re is the Reynolds number. Neglecting minor losses, the ratio of the frictional pressure drop in pipe 1 to that in pipe 2, $\left(\frac{\Delta P_1}{\Delta P_2}\right)$, is given by

a) $\left[\frac{d_2}{d_1}\right]^{5-n}$

b) $\left[\frac{d_2}{d_1}\right]^{5}$

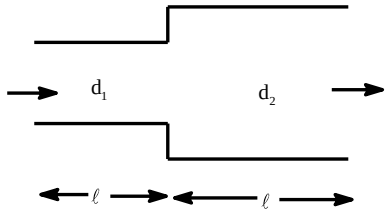
c) $\left[\frac{d_2}{d_1}\right]^{3-n}$

d) $\left[\frac{d_2}{d_1}\right]^{5+n}$

Answer: a

Explanation:

As pipes are in series, discharge will be same




$$\frac{\Delta P}{W}=\frac{fLQ^2}{12d^5}\Rightarrow \Delta P\propto\frac{f}{d^5}$$

$$\Delta P\propto\frac{K\,Re^{-n}}{d^5}\quad Re=\frac{\rho VD}{\mu}=\frac{VD}{\upsilon}$$

$$Q=\frac{\pi}{4}d^2V\Rightarrow V=\frac{4Q}{\pi d^2}$$

$$Re=\frac{d}{\upsilon}\frac{4Q}{\pi d^2}\Rightarrow Re\propto\frac{1}{d}$$

$$\Delta P\propto\frac{KPe^{-n}}{d^5}\Rightarrow\Delta P\propto\frac{1}{d^5}\left(\frac{1}{d}\right)^{-n}$$

$$=\frac{\Delta P_1}{\Delta P_2}=\left(\frac{d_1}{d_2}\right)^{n-5}\text{ or }\frac{\Delta P_1}{\Delta P_2}=\left(\frac{d_2}{d_1}\right)^{5-n}$$

36. Heat is generated uniformly in a long solid cylindrical rod (diameter = 10 mm) at the rate of $4\times10^7$ W/ $m^2$. The thermal conductivity of the rod material is 25 W/m K. Under steady state conditions, the temperature difference between the centre and the surface of the rod is_____$^0$C.

Answer: 10

Explanation:

$$T_C-T_S-\frac{q_gR^2}{4K}=\frac{4\times10^7\times\left(5\times10^{-3}\right)^2}{4\times25}=10\,K\text{ or }10^0C$$

37. Two cutting tools with tool life equations given below are being compared:

Tool 1: $VT^{0.1}=150$

Tool 2: $VT^{0.3} = 300$

Where V is cutting speed in m/minute and T is tool life in minutes. The breakeven cutting speed beyond which Tool 2 will have a higher tool life is _____m/minute.

**Answer:** 106

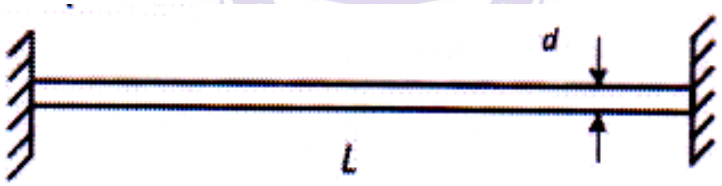
**Explanation:**

$$VT^{0.1} = 150$$

$$VT^{0.3} = 300$$

$$\frac{\left(VT^{0.1}\right)^3}{VT^{0.3}} = \frac{150^3}{300} \Rightarrow V^2 = \frac{150^2}{2} \Rightarrow V = 106 \text{ m/min}$$

38. An initially stress-free massless elastic beam of length L and circular cross-section with diameter d $(d << L)$ is held fixed between two walls as shown. The beam material has Young's modulus E and coefficient of thermal expansion $\alpha$. in the beam is slowly and uniformly heated, the temperature rise required to cause the beam to buckle is proportional to




a) d
b) $d^2$
c) $d^3$
d) $d^4$

**Answer:** b

**Explanation:**

$$F_{th} = F_{buckling}$$

$$\propto \Delta TEA = \frac{\pi^2 EI}{L^2}$$

$$\propto \Delta TE.\frac{\pi}{4}d^2 = \frac{\pi E.\pi d^4}{64\ L^2}$$

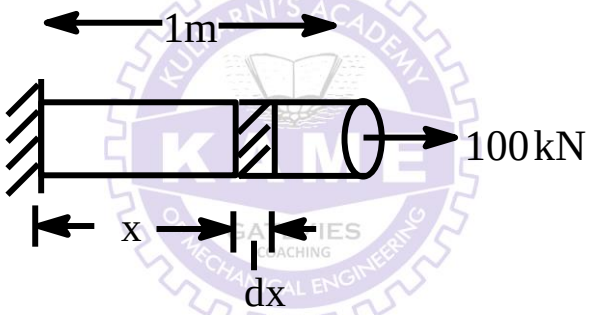
$$\Delta T = \left(\frac{\pi^2}{16L^2 \propto}\right) d^2$$

$$\Delta T \propto d^2$$

39. A horizontal bar, fixed at one end (x = 0), has a length of 1 m, and cross-sectional area of 100 mm². Its elastic modulus varies along its length as given by $E(x) = 100e^{-x}$ GPa, where x is the length coordinate (in m) along the axis of the bar. An axial tensile load of 10 kN is applied at the free end $(x = 1)$. The axial displacement of the free end is _____mm.

Answer: 1.718

Explanation:




$$\delta L = \frac{10 \times 10^3}{100 \times 100e^{-x} \times 10^3} dx$$

Total change in length $\int_0^1 \frac{e^x}{1000} dx$

$$\left|\frac{e^x}{1000}\right|_0^1 = \frac{1}{1000}(e-1)$$

$$= 1.718 \text{ mm}$$

40. Two disks A and B with identical mass (m) and radius (R) are initially at rest. They roll down from the top of identical inclined planes without slipping. Disk A has all of its mass concentrated at the rim, while Disk B ahs its mass uniformly distributed. At the

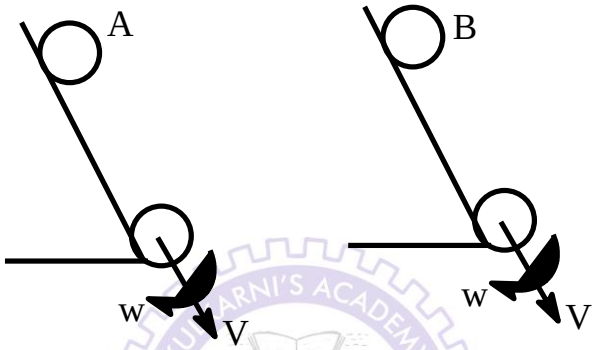
bottom of the plane, the ratio of velocity of the center of disk A to the velocity of the center of disk B is

a) $\sqrt{\frac{3}{4}}$ b) $\sqrt{\frac{3}{2}}$

d) 1 d) $\sqrt{2}$

Answer: a

Explanation:




$$mgh = \frac{1}{2}mV_A^1 + \frac{1}{2}mR^2\frac{V_A^2}{R^2} \qquad mgh = \frac{1}{2}mV_B^2 + \frac{1}{2}\times\frac{1}{2}mR^2.\frac{V_B^2}{R^2}$$

$$mgh = mV_A^2 \qquad mgh = \frac{3}{4}mV_B^2$$

$$V_A = \sqrt{gh} \qquad V_B = \sqrt{\frac{4}{3}gh}$$

$$\frac{V_A}{V_B} = \frac{\sqrt{gh}}{\sqrt{\frac{4gh}{3}}} = \sqrt{\frac{3}{4}}$$

41. The velocity profile inside the boundary layer for flow over a flat plate is given as $\frac{u}{u_\omega} = -\sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$, where $U_\infty$ is the free stream velocity $\delta$ is the local boundary layer thickness. If $\delta^*$ is the local displacement thickness, the value of $\frac{\delta^*}{\delta}$ is

a) $\frac{2}{\pi}$ b) $1-\frac{2}{\pi}$

c) $1+\frac{2}{\pi}$ d) 0

Answer: b

Explanation:

$$\frac{u}{u_\infty}=\sin\left(\frac{\pi y}{2\delta}\right)$$

$$\delta^*=\int_0^\delta\left(1-\frac{u}{u_\infty}\right)dy$$

$$\delta^*=\int_0^\delta\left(1-\sin\frac{\pi y}{2\delta}\right)dy$$

$$\delta^*=\left[y+\frac{\cos\frac{\pi y}{2\delta}}{\frac{\pi}{2\delta}}\right]_0^\delta$$

$$\delta^*=\left[y+\frac{2\delta}{\pi}\cos\frac{\pi y}{2\delta}\right]_0^\delta$$

$$\delta^*=\delta+\frac{2\delta}{\pi}(0)-0-\frac{2\delta}{\pi}$$

$$\delta^*=\delta-\frac{2\delta}{\pi}\Rightarrow\frac{\delta^*}{\delta}=1-\frac{2}{\pi}$$

42. P (0, 3), Q (0.5, 4), and R (1, 5) are three points on the curve defined by $f(x)$. Numerical integration is carried out using both Trapezoidal rule and Simpson's rule within limits x = 0 and x = 1 for the curve. The difference between the two results will be

a) 0 b) 0.25 c) 0.5 d) 1

Answer: a

Explanation:

Given points on the curve f(x) are

$P(0,3), Q(0.5,4) \& R(1,5)$

Trapezoidal rule $= \frac{h}{2}\left[(y_0 + y_2) + 2y_1\right]$

$= \frac{0.5}{2}\left[(3+5) + 2(4)\right] = 4$

Simpsons rule $= \frac{h}{3}\left[(y_0 + y_2) + 4y_1\right]$

$= \frac{0.5}{3}\left[(3+5) + 4(4)\right] = 4$

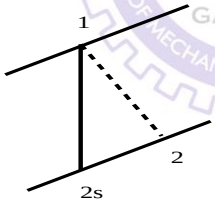
Difference $= 4 - 4 = 0$

43. The pressure ratio across a gas turbine (for air, specific heat at constant pressure, $c_p = 1040\,J/kg.K$ and ratio of specific heats, $\gamma = 1.4$) is 10. If the inlet temperature to the turbine is 1200 K and the isentropic efficiency is 0.9, the gas temperature at turbine exit is_____K.

Answer: 679.38 K

Explanation:

$T_1 = 1200\,K$




$\frac{T_1}{T_{2S}} = \left(\frac{P_1}{P_{2s}}\right)^{\frac{\gamma-1}{\gamma}}$

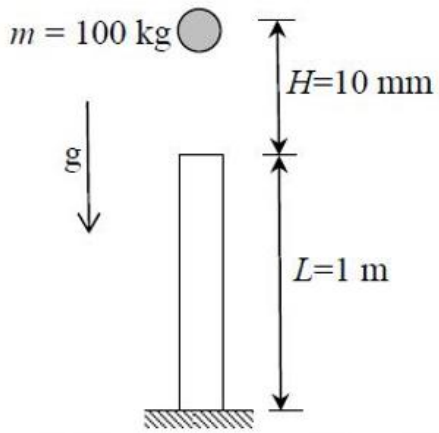
$\frac{1200}{T_{2s}} = (10)^{\frac{1.4-1}{1.4}}$

$T_{2s} = 621.537\,K$

$\eta_T = \frac{T_1 - T_2}{T_1 - T_{2s}} \Rightarrow 0.9 = \frac{1200 - T_2}{1200 - 621.537}$

$T_2 = 679.38\,K$

44. A point mass of 100 kg is dropped onto a massless elastic bar (cross-sectional area = 100 mm$^2$, length = 1m, Young's modulus = 100 GPa) from a height H of 10 mm as shown (Figure is not to scale). If g = 10 m/s$^2$, the maximum compression of the elastic bar is_____mm.




Answer: 1.514

Explanation:

Let the compression be x m

From the conservation of energy.

PE = Elastic energy

$$mg(h+x)=\frac{1}{2}Kx^2$$

$$K=\frac{EA}{l}$$

$$mg(h+x)=\frac{1}{2}\frac{EA}{\ell}.x^2$$

$$100\times10(0.01+x)=\frac{1}{2}\times\frac{100\times10^9\times100\times10^{-6}}{\ell}\times x^2$$

$$10+1000x=5\times10^6x^2$$

$$5\times10^6x^2-1000x-10=0$$

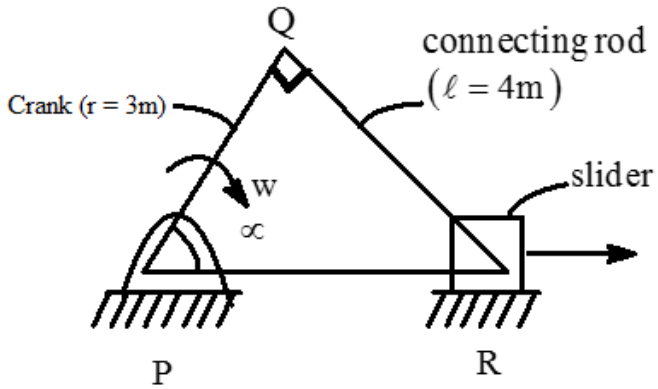
$$5\times10^5x^2-100x-1=0$$

Solving, $x=1.514\times10^{-3}\,m=1.514\,mm$

45. For an inline slider-crank mechanism, the lengths of the crank and connecting rod are 3m and 4m, respectively. At the instant when the connecting rod is perpendicular to the crank, if the velocity of the slider is 1 m/s, the magnitude of angular velocity (upto 3 decimal points accuracy) of the crank is_____radian/s.

**Answer:** 0.266

**Explanation:**




$V_{slider} \times \sin \propto = wr$

$\tan \propto = \frac{4}{3} \Rightarrow \propto = 53.13^0$

$1 \times \sin 53.13 = w \times 3$

$\omega = 0.266$ rad./sec

46. For the vectors $\vec{V} = 2yz\hat{i} + 3xz\hat{j} + 4xy\,k$, the value of $\nabla.\left(\nabla \times \vec{V}\right)$ is_____.

**Answer:** 0

**Explanation:**

$$\nabla \times \vec{V} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2yx & 3xz & 4xy \end{vmatrix} = x\hat{i} - 2y\hat{j} + z\hat{k}$$

$$\nabla\left(\nabla \times \vec{V}\right) = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(-2y) + \frac{\partial}{\partial z}(z)$$

$$= 1 - 2 + 1 = 0$$

47. One kg of an ideal gas (gas constant, R = 400 J/kg.K · specific heat at constant volume, $c_v = 1000\ j/Kg.K$) at 1 bar, and 300 K is contained in a sealed rigid cylinder. During an adiabatic process, 100 kJ of work is done on the system by a stirrer. The increase in entropy of the system is_____J/K.

Answer: 287.68

Explanation:

From first law, $Q = du + W$

$Q = 0$ (Adiabatic Process)

$du + W = 0 \Rightarrow -W = du$

Stirrer work = work done on the system

$-(-100) = du \Rightarrow du = 100\,kJ$

$mCv\ (T_2 - T_1) = 100$

$1 \times 1 \times (T_2 - T_1) = 100$

$(T_2 - 300) = 100 = T_2 = 400\ K$

$$S_2 - S_1 \left( C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \right) m$$

Rigid container $\Rightarrow V_1 = V_2$

$$S_2 - S_1 = mC_v \ln \frac{T_2}{T_1} = 1 \times 1000 \times \ln \left( \frac{400}{300} \right)$$

$S_2 - S_1 = 287.68\,J/k$

48. A block of length 200 mm is machined by a slab milling cutter 34 mm in diameter. The depth of cut and table feed are set at 2 mm and 18 mm/minute, respectively. Considering the approach and the over travel of the cutter to be same, the minimum estimated machining time per pass is_____minutes.

Answer: 12

Explanation:

Approach = over travel (given)

$= \sqrt{2(34-2)} = 8\text{mm}$

Length of tool travel = 200 + 8 + 8 = 216 mm

$$\text{Estimated time} = \frac{\text{length of tool travel}}{\text{feed}} = \frac{216}{18}$$

= 12 mm

49. Circular arc on a part profile is being machined on a vertical CNC milling machine. CNC part program using metric units with absolute dimensions is listed below·

---

N60 G01 X 30 Y 55 Z – 5 F50
N70 G02 X 50 Y 35 R 20
N80 G01 Z 5

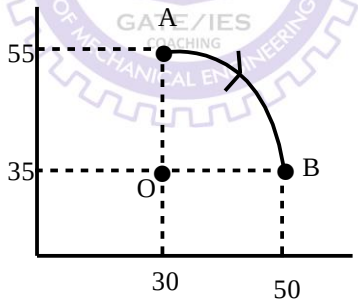
---

The coordinates of the centre of the circular arc are·

a) $(30,55)$ b) $(50,55)$ c) $(50,35)$ d) $(30,35)$

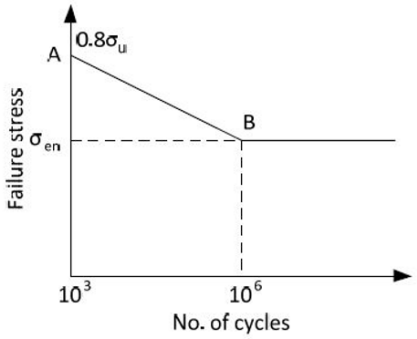
Answer: d

Explanation:




centre $(30,35)$

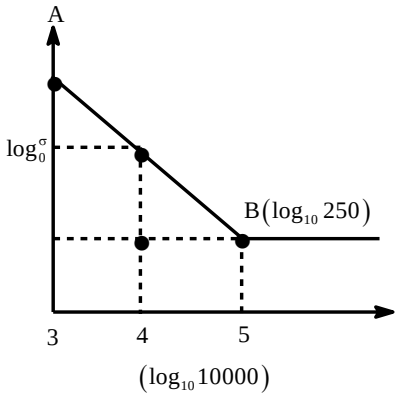
50. A machine element has an ultimate strength $(\sigma_u)$ of 600 N/mm², and endurance limit $(\sigma_{en})$ of 250 N/mm². The fatigue curve for the element on a log-log plot is shown below. If the element is to be designed for a finite life of 10000 cycles, the maximum amplitude of a completely reversed operating stress is_____N/mm².




**Answer:** 386.19

**Explanation:**

$Log_{10}\sigma$ Vs $Log_{10}$ (Number of cycles)




From similar triangles, $\sigma = 386.19\ N/mm^2$

51. Two models, P and Q, of a product earn profits of Rs.100 and Rs.80 per piece, respectively. Production times for P and Q are 5 hours and 3 hours, respectively, while the total production time available is 150 hours. For a total batch size of 40, to maximize profit, the number of units of P to be produced is_____.

**Answer:** 15

**Explanation:**

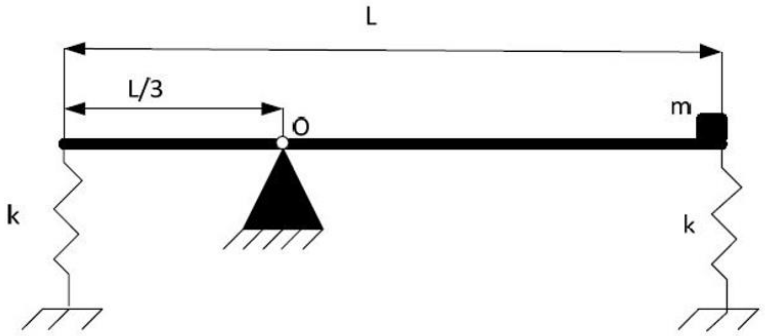
$Max\ Z = 100P + 80Q$

$5P + 3Q \le 150$

V

By graphical approach, P = 15; Q = 25

52. A thin uniform rigid bar of length L and mass M is hinged at point O, located at a distance of $\frac{L}{3}$ from one of its ends. The bar is further supported using spring, each of stiffness k, located at the two ends. A particle of mass $m=\frac{M}{4}$ is fixed at one end of the bar, as shown in the figure. For small rotations of the bar about O, the natural frequency of the system is




a) $\sqrt{\frac{5k}{M}}$

b) $\sqrt{\frac{5k}{2M}}$

c) $\sqrt{\frac{3k}{2M}}$

d) $\sqrt{\frac{3k}{M}}$

Answer: b

Explanation:

$$\tau_{restoring}=K\times\frac{2L}{3}\times\theta\times\frac{2L}{3}+K\times\frac{L}{3}\times\theta\times\frac{L}{3}$$

$$=\left(\frac{5KL^2}{9}\right)\theta$$

$$\left\{\frac{ML^2}{12}+M\left(\frac{L}{6}\right)^2+\frac{M}{4}\left(\frac{2L}{3}\right)^2\right\}\propto=\left(\frac{5KL^2}{9}\right)\theta$$

$$\frac{8ML^2}{36^0}\propto=\frac{5KL^2}{9}\theta$$

$$\propto=\frac{5K}{2M}\theta$$

$$w = \sqrt{\frac{5K}{2M}}$$

53. A 10 mm deep cylindrical cup with diameter of 15 mm is drawn from a circular blank. Neglecting the variation in the sheet thickness, the diameter (upto 20 decimal points accuracy) of the blank is_____mm.

Answer: 28.7

Explanation:

$h = 10; d = 15\,mm$

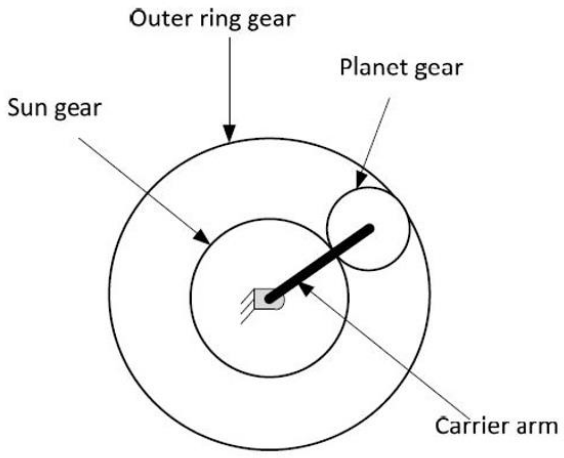
$$\frac{\pi}{4}D^2 = \frac{\pi}{4}d^2 + \pi dh.4$$

$$D^2 = d^2 + 4dh$$

$$D^2 = 15^2 + 4(15)(10)$$

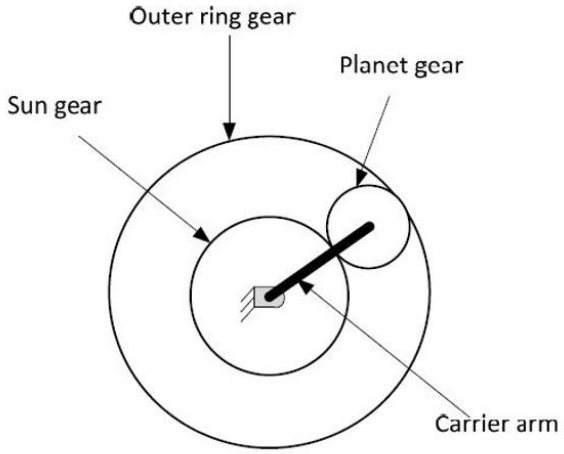
$$D = 28.7\,mm$$

54. In an epicyclic gear train, shown in the figure, the outer ring gear is fixed, while the stin gear rotation counterclockwise at 100 rpm. Let the number of teeth on the sun, planet and outer gears to be 40, 25, and 100, respectively. The ratio of magnitudes of angular velocity of the planet gear to the angular velocity of the carrier arm is_____.




Answer: 3

Explanation:




| | $N_b$ | $N_P$ | $N_0$ | $N_{arm}$ |
|---|---|---|---|---|
| Let sun gear be rotated by x rpm while arm is fixed (counter) | x | $-2x$ | $-\frac{x}{2}$ | 0 |
| Arm be rotated at y | $x+y$ | $-2x+y$ | $-\frac{x}{2}+y$ | y |

$$x+y=100;-\frac{x}{y}+y=0$$

$$x=\frac{200}{y};y=\frac{100}{3}$$

$$N_P=-2x+y=-2\left(\frac{200}{3}\right)+\frac{100}{3}=-100$$

$$N_{arm}=y=\frac{100}{3}$$

$$\frac{N_P}{N_{arm}}=\frac{|N_P|}{|N_{arm}|}=\frac{100}{\frac{100}{3}}=3$$

55. A sprue in a sand mould has a top diameter of 20 mm and height of 200 mm. The velocity of the molten meal at the entry of the sprue is 0.5 m/s. Assume acceleration due to gravity as 9.8 m/s$^2$ and neglect all losses. If the mould is well ventilated, the velocity (upto 3 decimal points accuracy) of the molten metal at the bottom of the sprue is____m/s.

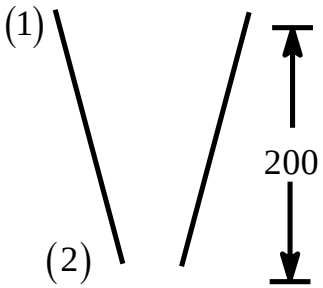
**Answer:** 2.043

**Explanation:**

$$A_1V_1 = A_2V_2$$

$$\phi = 20$$

$$V_2 = 0.5\,m/s$$




$$\frac{\pi}{4}(0.02)^2 \times 0.5 = A_2V_2$$

$$\frac{V_1^2}{2g} + 0.2 = \frac{V_2^2}{2g}$$

$$V_2 = 2.043\,m/s$$



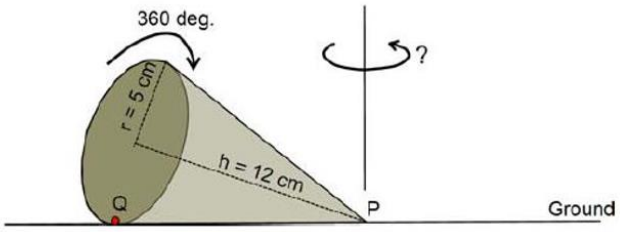
## SECTION: GENERAL APTITUDE

**01.** He was one of best_____and I felt his loss_____

a) friend, keenly b) friends, keen c) friend, keener d) friends, keenly

Answer: d

02. A right –angled cone (with base radius 5 cm and height 12 cm), as shown in the figure below, is rolled on the ground keeping the point P fixed until the point Q (at the base of the cone, as shown touches the ground again.




By what angle (in radians) about P does the cone travel?

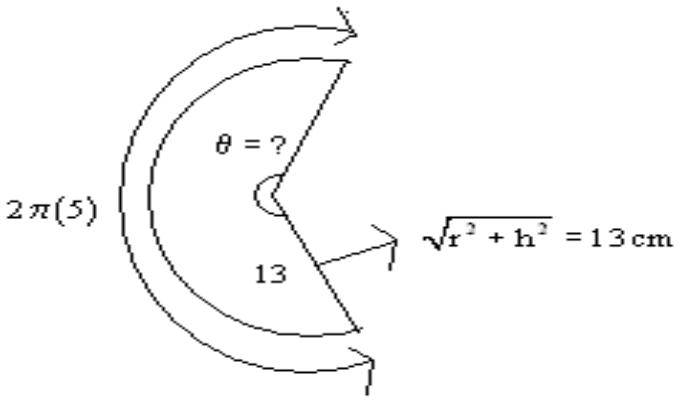
a) $\frac{5\pi}{12}$ b) $\frac{5\pi}{24}$

c) $\frac{24\pi}{5}$ d) $\frac{10\pi}{13}$

Answer: d

Explanation:

According to the given information about the figure, the new shape is as follows




Required angle

$$\theta = \frac{1}{R} = \frac{2\pi(5)}{13} = \frac{10\pi}{13}$$

03. P, Q, and R talk about S's car collection. P states that S has at least 3 cars. Q believes that S has less than 3 cars. R indicates that to his knowledge, S has at least one car. Only one of P, Q and R is right. The number of cars owned by S is

a) 0 b) 1 c) 3 d) Cannot be determined

Answer: a

04. As the two speakers became increasingly agitated, the debate became_____.

a) luckewarm b) poetic c) forgiving d) heated

Answer: d

05. In a company with 100 employees, 45 earn Rs.20, 000 per month, 25 earn Rs.30, 000, 20 and earn Rs.40, 000, 8 earn rs.60, 000, and 2 earn Rs.150, 000. The median of the salaries is

a) Rs.20, 000 b) Rs.30, 000 c) Rs.32, 300 d) Rs.40, 000

Answer: b

06. What is the sum of the missing digits in the subtraction problem below?

$$\begin{array}{r} 5\text{---} \\ -48\_89 \\ \hline 1111 \end{array}$$

a) 8 b) 10

c) 11 d) cannot be determined

Answer: a

Explanation:

From subtraction procedure, it is clear to identify the missing digit as

$$\begin{array}{r} 5\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0} \\ -4\ 8\ \underline{8}\ 8\ 9 \\ \hline 1\ 1\ 1\ 1 \\ \hline \end{array}$$

07. Let S1 be the plane figure consisting of the points (x, y) given by the inequalities $|x-1| \le 2$ and $|y+2| \le 3$. Let S2 be the plane figure given by the inequalities $x - y \ge -2$, $y \ge 1$, and $x \le 3$. Let S be the union of S1 and S2. The area of S is

a) 26 b) 28

c) 32 d) 34

Answer: c

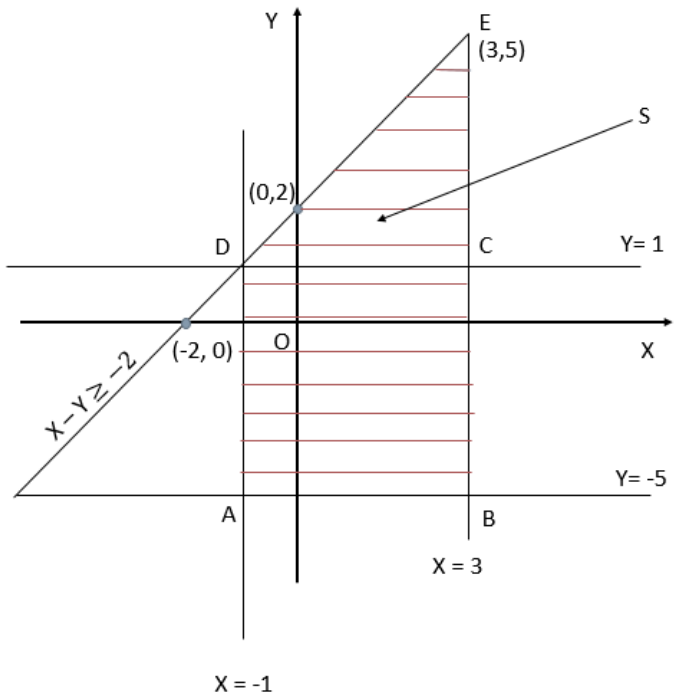
Explanation:

$S_1 \rightarrow |X-1| \le 2 \rightarrow -2 \le X-1 \le 2 \rightarrow -1 \le X \le 3$

$|Y+2| \le 3 \rightarrow -3 \le Y+2 \le 3 \rightarrow -5 \le Y \le 1$

$S_2 \rightarrow X - Y \ge -2 \; ; \; Y \ge 1 \; and \; X \le 3$

Since $S_1 \cup S_2$ , Area of S is as shown below




Required area S = Area of ABCD + Area of $\Delta\, CDE$

$= \{4 \times 6\} + \{\frac{1}{2} \times 4 \times 4\}$

$= 24 + 8 = 32$ units

08. Two very famous sportsmen Mark and Steve happened to be brothers, and played for country K. Mark teased James, an opponent from country E, "There is no way you are good enough to play for your country." James replied, "Maybe not, but at least I am the best player in my own family." Which one of the following can be inferred from this conversation?

a) Mark was known to play better than James

b) Steve was known to play better than Mark

c) James and Steve were good friends

d) James played better than Steve

Answer: b

09. Here, throughout the early 1820s, Stuart continued to fight his losing battle to allow his sepoys to wear their caste-marks and their own choice of facial hair on parade, being again reprimanded by the commander-in-chief. His retort that 'A stronger instance than this of European prejudice with relation to this country has never come under my observations' had no effect on his superiors." According to this paragraph, which of the statements below is most accurate?

a) Stuar's commander-in-chief was moved by this demonstration of his prejudice.

b) The Europeans were accommodating of the sepoys desire to wear their caste-marks.

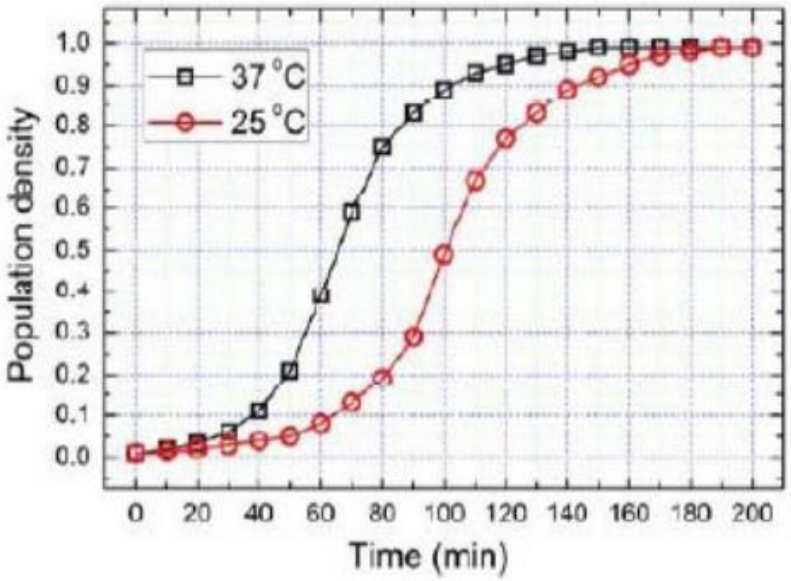
c) Stuart's 'losing battle' refers to his inability to succeed in enabling sepoys to wear caste-marks.

d) The commander-in-chief was exempt from the European prejudice that dictated how the sepoys were to dress.

Answer: c

10. The growth of bacteria (lactobacillus) in milk leads to curd formation. A minimum bacterial population density of 0.8 (in suitable units) is needed to form curd. In the

graph below, the population density of lactobacillus in 1 liter of milk is plotted as a function of time, at two different temperatures, 25°C and 37°C.




Consider the following statements based on the data shown above·

I. The growth in bacterial population stops earlier at 37°C as compared to 25°C

II. The time taken for curd formation at 25°C is twice the time taken at 37°C

which one of the following options is correct?

a) Only I

b) Only II

c) Both I and II

d) Neither I nor II

Answer: a